S-95
1965

Annual

Sherwood Theoretical Meeting

April 22-23, 1965

Princeton, New Jersey

ABSTRACTS

Annual Sherwood Theoretical Meeting

Princeton, New Jersey

April 22-23, 1965

Engineering Quadrangle, Princeton University

Session I
Thursday Morning at 9:30

Convocation Room (C 217)
(J. L. Johnson presiding)

D. Dobrott	Exponentially Damped Modes in the Presence of Magnetic Field Shear
A. Kadish	Stability Criteria for Guiding Center Plasma
S. Yoshikawa	Theoretical Problems Associated with Ion Cyclotron Heating
F. C. Hoh	Quasi-Linear Theory for Small Larmor Radius Universal Instabilities
G. A. Pearson A. N. Kaufman	Stabilization of a Current-Carrying Plasma by Wave-Particle Interaction
B. D. Fried S. L. Ossakow	The Kinetic Equation for an Unstable Plasma in Parallel Electric and Magnetic Fields
O. Buneman	Interstreaming Instabilities in the Presence of Binary Collisions
O. Buneman	Studies of an Electrostatic Nonhelical Mode of Anomalous Diffusion

Session II
Thursday Afternoon at 2:00

Lecture Room (C 207)
(S. Buchsbaum presiding)

R. Sagdeev	On Galeev's Nonlinear Theory of Rosenbluth-Post 'Loss Cone' Stability
J. Killeen K. J. Whiteman	The Computation of Hydromagnetic Equilibria with Finite Pressure in Minimum-B Systems

M. N. Rosenbluth	Flute Type Instabilities of Loss Cone Velocity Space Distributions
J. L. Johnson	The Negative V'' Stellarator
R. M. Kulsrud	The Resistive Ballooning Mode in Negative V'' Stellarators
C. S. Gardner	Toroidal Vacuum Field with Minimized Variation of $\oint d\ell/B$
J. E. Drummond B. Ancker-Johnson	The Possible Value of Electron-Hole Plasma Experiments as Tests of Fusion Containment Schemes

Session III

Friday Morning at 9:30

Lecture Room (C 207)

(I. B. Bernstein presiding)

N. Christofilos	Alternating Gradient Field of the E-layer → Minimum B everywhere(?)
T. G. Northrop C. S. Liu	The Second Adiabatic Invariant in the Second Approximation
A. McMann	Finite Gyration Radius Corrections to the Moment Equations
M. Kruskal J. B. Taylor	Variational Principle for Long-Time Equilibrium in Particle Description
R. E. Aamodt D. L. Book	Influence of Wave Reflection on Mirror Confinement
H. Grad	Stability of Mirror Machines
H. P. Furth	Instabilities of the "Low-Density Stable Regime" in the Mirror Machine
N. A. Krall L. D. Pearlstein	Stability of Flute and Universal Modes in a Finite Pressure Plasma
X T. K. Fowler Y. Shima	Flute Instabilities at Ion Gyrofrequency

Session IV
Friday Afternoon at 2:00

Lecture Room (C 207)
(J. Berkowitz presiding)

W. Drummond	Recent Experimental and Theoretical Work in the Soviet Union
B. Bertotti A. Cavaliere P. Guipponi	Ion Waves in Bounded Plasma
H. Kever G. Morikawa	Steady Nonlinear Waves in a Warm Collision-free Plasma
H. Weitzner D. Dobrott	Ion-Acoustic Waves and Electron Plasma Oscillations
P. N. Hu	Structure of a Perpendicular Shock Wave
T. J. Birmingham	Radiation by a Plasma: Quadrupole Bremsstrahlung and Synchrotron Emission
O. C. Eldridge E. G. Harris	The Diffusion of Test Particles in a Magnetic Field

Exponentially Damped Modes in the Presence of Magnetic Field Shear

D. Dobrott

Courant Institute of Mathematical Sciences
New York University

Electrostatic waves are examined in an inhomogeneous plasma in which a slight shear exists in the magnetic field. The shear and inhomogeneity are produced by a small uniform macroscopic drift in the direction of a strong uniform magnetic field. The resulting configuration is that of the self-consistent Harris sheet model,[1] "stabilized" by a strong magnetic field. The analysis is made within the framework of the linearized Vlasov equation. There are two smallness parameters in the problem: the macroscopic drift and the reciprocal of the applied magnetic field. Use is made of these parameters and the high degree of symmetry to develop approximate orbits by the method of Krylov and Bogoliubov.[2] The resulting orbits are not uniformly valid in time, and hence results derived are only asymptotic. However, by using the WKB method, certain low- and gyro-frequency modes are found to be exponentially damped of a finite period of time and thereafter dispersed more slowly. The low-frequency mode is of the "universal" or "drift-wave" type discussed by Rudakov and Sagdeev[3] as well as Krall and Rosenbluth.[4]

---

1. E. G. Harris, "On a Self-Consistent Field Method for a Completely Ionized Gas", NRL Report 4944, Naval Research Laboratory, Washington, D. C. (1957).

   Selin, Kovrizhnik

2. N. Krylov and N. N. Bogoliubov, Introduction to Non-Linear Mechanics, (Princeton University Press, Princeton, N. J., 1947).
3. L. I. Rudakov and R. Z. Sagdeev, Soviet Phys. —JETP 10, 952 (1960).
4. N. A. Krall and M. N. Rosenbluth, Phys. Fluids 4, 2 (1961).

Stokes lines

Sarel & Pellat

$K \int_{z}^{z_2} k_x dx = n + \frac{1}{2}$

remote turning points

$k_x^2 = \frac{\partial D}{\partial x}$

Airy functions

close turning points

$k_x^2 = \frac{1}{2} \frac{\partial^2 D}{\partial x^2}$

parabolic ~~diff.~~ cylindrical function

Stability Criteria for Guiding Center Plasma

Abraham Kādish

Courant Institute of Mathematical Sciences
New York University

A two-fluid, collisionless, space-filling plasma in a strong magnetic field is assumed, with particle orbits given by the lowest order guiding center theory.[1] The mathematical model used to describe the system is that derived by H. Grad,[2] in which the distribution functions are taken as functions of the magnetic moment per unit mass and the particle velocity along the magnetic field line as well as space and time. After linearization about a charge-neutral, homogeneous steady-state configuration, criteria for the stability of this configuration with respect to small initial perturbations are derived using analytic function theory and transform techniques. Under rather general conditions, it is shown that two necessary, but not sufficient, conditions for stability are that

$$\frac{B^2}{\mu_o} + P_\perp > P_\parallel \tag{1}$$

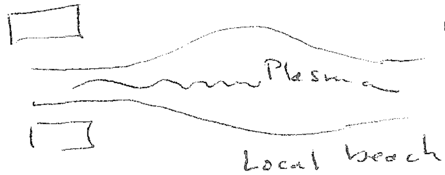
$$\frac{B^2}{\mu_o} + 2P_\perp > \frac{1}{3}\,\frac{P_\perp^2}{P_\parallel}\,\eta \tag{2}$$

where η is a functional of the distributions. It is shown that η is greater than unity for distributions which are singly peaked in the velocity parallel to the magnetic field and is equal to six for spherically symmetric and two-temperature Maxwellian distributions. In the latter case, (1) and (2) are, therefore, both trivially satisfied. Using a fluid model R. Lüst[3] had derived these conditions with $\eta = 1$. An easily verifiable sufficient condition for stability is derived, and it is shown that when exponential instabilities are present, the initial value problem for the linearized system is not well set. If, in the equilibrium

configuration, the distributions are bell shaped in the velocity parallel to $\vec{B}$, then inequalities (1) and (2) are necessary and sufficient for stability.

---

1. G. Hellwig, Z. Naturforschung 1, 508 (1955).
2. H. Grad, Proceedings of the Symposium on Electromagnetics and Fluid Dynamics of Gaseous Plasma. Polytechnic Institute of Brooklyn, April 1965.
3. R. Lüst, "On the Stability of a Homogeneous Plasma with Non-isotropic Pressure", AEC Report TID-7582, November 1959.



← can open up mirror to get uniform heating around machine

$B = 33kg$

$n = 5\times10^{12}$

$T_{OH} \approx 150V$

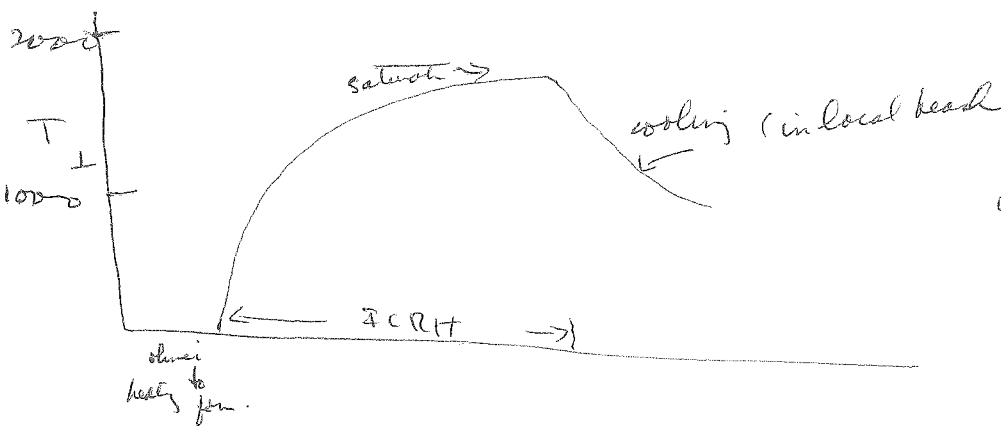
$\beta_{\parallel} \approx 5\times10^{-5}$

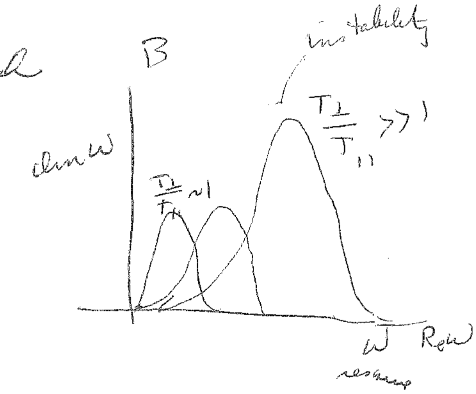
ICRH Power $\approx$ 200KW

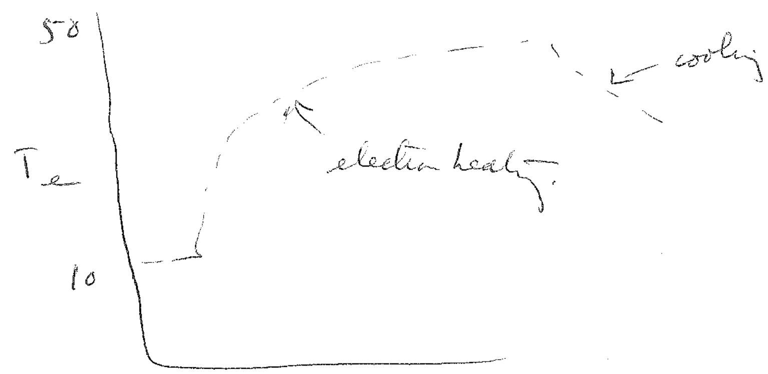
Beach volume 10^4 cm^3 (~.1 Total volume of Stellarator)

wave length grows shorter

hopes to study pumpout in absence of I_{OH}



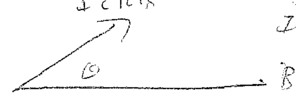




electron & ions are heated independently by waves - (many B - heating occurs at diff. field lines) not collisional - no dependence on turbulent heating - ruled out

so electrons also heated somewhat by wave - may help avoid certain instabilities

no heating of electrons is ∥ to ICRH wave vector



T_{e-i} cooling processes.

T_{n-i}

$$\frac{\omega}{k_z} \leq v_e$$

$$v_i \leq \frac{\omega_{ci}-\omega}{k_z}$$

Theoretical Problems Associated with
Ion Cyclotron Heating*

S. Yoshikawa

Plasma Physics Laboratory, Princeton University

Ion cyclotron heating[1] in the C stellarator yields ion temperatures of several thousand electron volts in the local mirrors (1/10 of the total volume) with the energy confinement time up to 1 msec. This confinement time can be accounted for by electron cooling and ion-ion scattering from the mirror. The apparent absence of influences of the MHD instability (e.g. ballooning) or the velocity anisotropy instability suggests that instabilities may not be too serious for moderate confinement times. When ions are heated in mirrors, the ratio of the perpendicular energy to the parallel energy increases. The wave energy absorption can be calculated to be proportional to $(1 - C\,\beta_{\parallel}^{1/3}\,T_{\perp}/T_{\parallel})$ if we assume that the plasma is Maxwellian with anisotropic temperature.[2] Here $\beta_{\parallel}$ is the parallel plasma pressure divided by the magnetic pressure, and C is a constant of the order of unity. Thus $T_{\perp}/T_{\parallel}$ cannot exceed $\beta_{\parallel}^{-1/3}$. This is consistent with the energy saturation observed experimentally. Direct heating of electrons by waves is also observed. This heating is independent of electron temperature suggesting that Landau damping of waves by electrons may be responsible. A similar observation with the whistler mode has been reported by Dolgopolov et al.[3]

---

* Work accomplished under auspices of U.S. Atomic Energy Commission.

1. T. H. Stix, The Theory of Plasma Waves (Mc Graw-Hill Book Co., New York, 1962).

2. For example, see Eq. (69), page 212 of reference 1.

3. V. V. Dolgopolov, A. I. Ermahov, N. I. Nagarov, K. N. Stepanov, and V. T. Tolok, Soviet Phys.—JETP 18, 866 (1964).

Problems
1. Ion Confinement in local mirrors
2. Electron heating by Landau damping
3. Ion Heating limit-

near resonance $\xi = \frac{\omega_i - \omega}{k_z v_i} \sim 1$

S.E. $\propto$ $1 - \xi^{2/3}\beta_{\parallel}^{1/3}\frac{T_\perp}{T_\parallel}$ Rosenbluth Harris

Know $\perp$ temps by conductivity & density profiles to some extent - can do Balmer continuously

Quasi-Linear Theory for Small Larmor Radius Universal Instabilities

F. C. Hoh

Boeing Scientific Research Laboratories

A small Larmor radius quasi-linear theory is developed for a class of unstable ion waves whose phase velocities along the magnetic field lie between the thermal velocities of ions and electrons. In the approximation of localized perturbations, the final state quantities of the plasma are given without specifying the detailed instability mechanism. An essential feature is that the quasi-linear theory predicts an anomalous "displacement" of a group of resonant particles rather than any anomalous diffusion. The theory is applied to a special type of the current-driven universal instability. Its quasi-linear consequences are explored.

Stabilization of a Current-Carrying Plasma by
Wave-Particle Interaction

Gary A. Pearson
University of California and
Lawrence Radiation Laboratory, Berkeley

and

Allan N. Kaufman

University of California, Los Angeles

The Lenard-Balescu equation has been solved for the electron velocity distribution in a uniform Coulomb ($\underline{B} = 0$) plasma, with $\theta_e/\theta_i >> 1$, subjected to a uniform static electric field E_o. It is found that the plasma remains stable as E_o increases well beyond the value E_{crit}, for which the Spitzer-Härm distribution predicts instability. This stabilization is due to a decreased anisotropy, caused by enhanced exchange of those waves which are barely stable and whose fluctuation energy is therefore large. Curves will be displayed of the enhanced Landau damping of these waves, and of the energy spectral density.

assumes
classical
Coulomb
unmagnetized
uniform
vary "slowly"
Ω suff. large
electron-proton plasma as in S-14
$\theta_e >> \theta_i$
$E_0 > E_{crit} << E_{run}$

assumes $f_i(v)$ is max
$f_e(v)$ is " $+ E_0 f_1(v,t) \cos\alpha$

$\left(\frac{\partial f}{\partial t}\right)_{\text{Landau F-P}}$

v, α, E_0

$E_0 << E_{runaway} = \frac{n_e e q_e}{\sigma_{Spitzer-Harm}}$

wave-particle interactions

The Kinetic Equation for an Unstable Plasma in Parallel Electric and Magnetic Fields

Burton D. Fried and Sidney L. Ossakow

University of California, Los Angeles

The response of a homogeneous plasma to the sudden application of a strong, uniform electric field was studied[1] by solving, self-consistently, the coupled equations for the one-particle and two-particle distribution functions, and it was shown that wave-particle interactions act to limit the electron runaway current, and, in fact, cause a deceleration of the mean electron velocity. This work has now been extended to include a very strong magnetic field, $\underline{B}_o$, parallel to the electric field. The basic approximations are the same as in the earlier work: neglect of three-particle correlations and the use of an adiabatic ansatz for the time dependence of the fluctuations. However, in the strong field limit, use of the Bernstein-Kruskal expansion in $k_\perp r_c$ ($k_\perp$ = wave number perpendicular to $\underline{B}_o$, r_c = ion cyclotron radius) makes the problem effectively one-dimensional, thus eliminating the need for factorization assumptions of the form $f(\underline{v}) = f_{\parallel}(v_{\parallel}) f_{\perp}(v_{\perp})$. The kinetic equation becomes a diffusion equation in velocity space, and only the dominant contributions to the diffusion coefficient, D, arising from the fastest growing waves, are retained. D is proportional to the equilibrium energy W of the fluctuations, but unlike the case $B_o = 0$ the calculation of W as a function of wave number presents certain difficulties. Unequal electron and ion temperatures are assumed, but it is necessary to do the calculations for a specific value of T_e/T_i, rather than simply assuming it to be large, as can be done when $B_o = 0$. For a number of parameter choices, W is calculated and the kinetic equation is solved as an initial value problem. From $f(v, t)$ the current is computed as a function of time. In contrast to the case $B_o = 0$, it is found that the diffusion is effective only for those electrons which have very small velocities (of the order of $(m/M)^{1/2}$) in the ion rest frame.

1. E. C. Field and B. D. Fried, Phys. Fluids $\underline{7}$, 1937 (1964).

Interstreaming Instabilities in the Presence
of Binary Collisions*

O. Buneman

Stanford University

Relaxation (Krook) terms are introduced into the Vlasov equations for electrons and ions to describe binary collisions. Mutual collisional interaction is taken into account by a relaxation term which pulls each distribution function towards a Maxwellian having the local instantaneous center-of-gravity drift of the two species.

Energy and momentum are conserved with these relaxation terms. In the presence of a dc electric field, a steady state is possible in which each species is non-Maxwellian and their relative drift represents the current associated with purely collisional resistivity. However, when the current or drift exceeds a certain limit close to the electron thermal velocity, this state becomes unstable, as it does in the absence of binary collisions. Broadly speaking, as long as the collision frequency lies below the plasma frequency, a plasma will not support fields so strong that electrons get accelerated to supersonic velocities in the time between collisions.

---

* Based on work by G. Reiter, with support from an AEC contract.

$\frac{\text{collision frequency}}{\omega_p} \sim 1$ plasma

~ 1 not a plasma

particles in Debye sphere ~ 1

Studies of an Electrostatic Nonhelical Mode of Anomalous Diffusion*

O. Buneman

Stanford University

Plasma columns which do not carry a direct or slowly varying current exhibit a type of anomalous diffusion that cannot be explained by helical instability theories. A possibly more violent instability seems to occur in radio-frequency and reflex discharges. It consists of $m = 1$, $m = 2$, and/or $m = 3$ flutes, with $k = 0$, rotating at the E/B speed compatible with measured potential differences between column and wall.

A two-dimensional model of the column within its insulating tube was programmed into a computer, ions and electrons being simulated as rods of electrostatically interacting charges.

Rotating charge bunches develop which create the azimuthal electric fields necessary for providing enhanced diffusion. The absence of a quiescent cylindrically symmetrical configuration (as in normal diffusion) can thus be understood as the result of electrostatic instabilities in crossed fields.

---

* Based on work by R. Hockney, with support from an ONR contract.

$k = 0 \Rightarrow$ nonhelical flutes
occurs without directed current (with electrons going in both directions)

The Computation of Hydromagnetic Equilibria with Finite Pressure in Minimum-B Systems*

John Killeen and Kenneth J. Whiteman+

Lawrence Radiation Laboratory
University of California, Livermore

The equations of hydromagnetic equilibria with a tensor pressure are

$$\vec{j} \times \vec{B} = \operatorname{div} \overline{\overline{p}}$$

$$\operatorname{curl} \vec{B} = \vec{j} \qquad\qquad \operatorname{div} \vec{B} = 0$$

If we make the assumption that $p_\perp$ and $p_\parallel$ are functions of B only and we consider only equilibria for which the field lines leave the plasma, e.g. mirror machines, stuffed cusp, etc., then the above equations reduce to the single equation

$$\operatorname{curl} \nu \vec{B} = 0 \tag{1}$$

where

$$\nu(B) = \frac{p_\parallel(B) - p_\perp(B)}{B^2} - 1 \tag{2}$$

In the axisymmetric case we can define the stream function $\psi(r, z)$ where

$$B_r = -\frac{1}{r}\frac{\partial \psi}{\partial z} \qquad\qquad B_z = \frac{1}{r}\frac{\partial \psi}{\partial r}$$

and Eq. (1) becomes

$$\frac{\partial}{\partial z}\left(\frac{\nu}{r}\frac{\partial \psi}{\partial z}\right) + \frac{\partial}{\partial r}\left(\frac{\nu}{r}\frac{\partial \psi}{\partial r}\right) = 0 \tag{3}$$

$$r\nu\, B_\theta = \text{constant}$$

We solve Eq. (3) numerically using difference methods with arbitrary functions $\nu(B)$. The procedure is to specify a vacuum field $\vec{B}_c$ and $p_\perp(B)$ and $p_\parallel(B)$ and then to compute the field due to the plasma and modify B. The solution is iterated many times until a self-consistent solution is obtained. We then increase the plasma pressure and repeat the procedure, thus obtaining solutions for a sequence of plasma pressures. We applied this program to the stuffed cusp configuration and the "disc" device of Furth[1] and Andreoletti.[2] Both of these are axisymmetric minimum-B systems. We have used $p_{\perp}(B)$ and $p_{\parallel}(B)$ of the type given by Taylor.[3] The effect of increasing the magnitude of the plasma pressure is to distort the shape of the well and to lower the minimum-B. In some cases the magnetic well is distorted sufficiently for the plasma to "spill out".

---

* Work performed under the auspices of the U.S. Atomic Energy Commission.

+ Culham Laboratory, Culham, Abingdon, Berks., England.

1. H. P. Furth, Phys. Rev. Letters 11, 308 (1963).
2. J. Andreoletti, Euratom Meeting on Mirror Confinement, July 1963 (unpublished).
3. J. B. Taylor, Phys. Fluids 6, 1529 (1963).

wave with $k_\perp >> k_\parallel$ - essentially unstable

∞ medium, B const., homogen, low β so $\varphi \sim e^{i(\omega t + k_y y + k_\parallel z)}$

Calculate disp relation (like Sagdeev's)

polarization drift electrodielectric — ordinary electrostatic plasma osc in B

$$0 = 1 + \frac{\omega_e^2}{\omega_c^2} - \frac{\omega_e^2}{\omega^2}\frac{k_\parallel^2}{k_\perp^2} + \text{ion contribution as Sagdeev}$$

if $\frac{\partial n}{\partial x} \neq 0$, picks $f_0 = f(v_\perp^2, v_\parallel^2)\left[1 + \varepsilon\left(x + \frac{v_y}{\Omega}\right)\right]$

now get for electrons $v_x = \frac{E_y}{B}$ $\qquad \varepsilon = \frac{1}{n}\frac{dn}{dx}$

set pileup of electron charge $v_x = \frac{\partial f_0}{\partial x}$ not balanced by ions

$$(kR_i)^2\left(\frac{\Omega_i^2}{\omega_{pi}^2} + \frac{m}{M}\right) - \frac{\varepsilon R_i}{y} + i\gamma y = 0$$

for small $\frac{\omega}{k_\perp v}$ this is first term of Sagdeev's F

$y = \frac{\omega}{k_\perp v_i}$ $\quad k_\parallel = 0$ $\quad (\gamma \sim 1 \quad (\text{tail of dist})$

quite loss come in diff has this form - can find unstable root

$$\omega = \frac{\Omega_i\,(\varepsilon R_i)^{3/4}}{\left(\frac{\Omega_i^2}{\omega_{pi}^2} + \frac{m}{M}\right)^{1/2}}$$

fastest growing mode all 3 terms are same order

For stability $\varepsilon R_i < \delta\left(\frac{\Omega_i^2}{\omega_{pi}^2} + \frac{m}{M}\right)^{2/3}$

$\frac{R}{R_i} \sim \left(\frac{M}{m}\right)^{2/3}$ $\quad \delta \sim 1$, depends on details of dist.

with finite $k_\parallel$ (convective modes) — waves will run along machine & be absorbed at ends

non-convective modes $\frac{R}{R_i} > \left(\frac{M}{m}\right)^{2/3}$

convective $\frac{L}{R_L} < 100$

⇒ relatively short fat object

~ very insensitive to shear (looks like Mikh.-Timofeev - 1/2 power - for case without loss cone - depended on very sharp resonance Ω_i, $k_y v_D$ - so could stabilize with shear)

now with shear effectively $k^2 \to k^2 [1 + \frac{L^2}{L^2_{shear}}]$ does nothing to stop instability

Flute Type Instabilities of Loss Cone Velocity Space Distributions*

Marshall N. Rosenbluth

General Atomic Division
General Dynamics Corporation

and

University of California, San Diego

We consider the problem of a plasma with a loss cone type velocity distribution and a radial density gradient. In addition to the convective modes discussed earlier[1] the radial density gradient makes possible the appearance of flute type modes which are nonconvective and are driven by the potential instability of the ion distribution function to the two-stream instability at frequencies above the ion cyclotron frequency. We find the waves to be unstable if

$$R_i \frac{1}{n} \frac{dn}{dx} > \left(\frac{m}{M}\right)^{2/3} \left(1 + \frac{B^2}{4\pi n m c^2}\right)^{2/3}$$

and relatively insensitive to longitudinal variation, shear, etc.

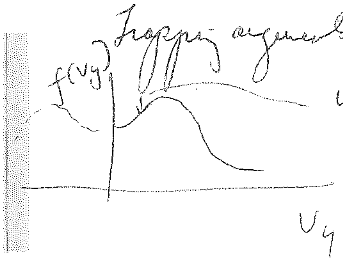
if cold plasma so effective line tying or if effective end-tying could stabilize - could have this in U bends etc where loss cone - in open-ended machines nothing to stabilize

---

* Work performed under the auspices of the U. S. Atomic Energy Commission at the University of California, and under a joint General Atomic-Texas Atomic Energy Research Foundation program on controlled thermonuclear reactions.

1. M. N. Rosenbluth and R. F. Post, GA-5764 (submitted to the Physics of Fluids).

Hopping arguments — if nonmonotonic dist. in one direction potentially unstable



unstable in this region $f(v_\perp^2, v_\parallel^2)$ near $v_y = 0$ have positive slope & possibility of instability (if you can find wave propagating with this kind of phase velocity)

$\omega > \Omega_i$ but $\omega < \Omega_e$ — electrons behave as in MHD - tied to B - ions don't know they are in magnetic field so can use straight line orbits — two stream (ions not gyrating) - treat as if not in B -

The Negative V'' Stellarator*

John L. Johnson+

Plasma Physics Laboratory, Princeton University

A series of numerical computations of the magnetic lines of force in a superposition of vacuum multipolar fields has been made to determine the efficacy of theoretical analyses which utilize the usual small-amplitude asymptotic expansions.[1,2] These computations show that: (1) the expansion results provide a good qualitative but not quantitative description of the magnetic surfaces even when the sizes of the parameters are far from the assumed ordering; and (2) numerical solutions of the equations for the lines of force can be used to determine the magnetic surfaces and obtain the equilibrium functions $V'(\psi)$ and $\iota(\psi)$.

---

* This work accomplished under the auspices of the U. S. Atomic Energy Commission.

+ On loan from Westinghouse Research Laboratory.

1. J. L. Johnson, Phys. Fluids 7, 2015 (1964).

2. J. L. Johnson, Princeton University Plasma Physics Laboratory MATT-Q-22 (April 1965), p. 217.

$\delta B \neq 0$

The Resistive Ballooning Mode in Negative V'' Stellarators*

Russell M. Kulsrud

Plasma Physics Laboratory, Princeton University

The stability of the negative V'' systems of Johnson[1] against the ballooning mode is investigated by the normal mode equations with zero and finite resistivity. For the case of zero resistivity a critical value of β, β_c, below which the systems are stable (where $\beta = p/(B^2/8\pi)$ evaluated at the center) is determined by an expansion technique. This value of β_c is

$$\beta_c = \frac{g}{2} \, \frac{U}{\Delta U} \, \frac{\pi^2 b^2}{L^2} \tag{1}$$

where $U = \int dl/B$ taken along an arbitrary length L of a line of force, ΔU is the variation of U from the line of force of maximum pressure to a line at the edge of the plasma; b is the nearest distance of the latter line from the axis and the length of line should be chosen to make the expression on the right-hand side of Eq. (1) as small as possible. g is a constant which in general is near one and depends on the particular configuration.

For the case of finite resistivity η the plasma is always unstable to the resistive ballooning mode, and if one can neglect the flow of plasma along the line of force, its growth rate is $\sigma_R = m^2/\beta_c \, \tau_{class}$ where $\tau_{class} = 4\pi a^2/\eta$.

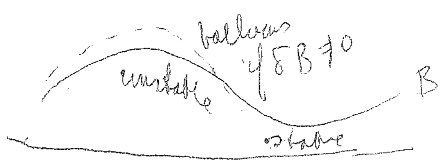
$\frac{2a}{R}$ well depth of U

well depth

---

* This work accomplished under the auspices of the U. S. Atomic Energy Commission.

1. J. L. Johnson, Phys. Fluids 7, 2015 (1964).



σ = growth rate

$\frac{1}{\sigma} > \frac{\pi L}{V_A}$ Walen - if the field line doesn't know it is tied

Similar to line tied interchange.

Toroidal Vacuum Field with Minimized
Variation of $\oint d\ell / B$ *

C. S. Gardner

Plasma Physics Laboratory, Princeton University

Analysis of the ballooning mode indicates that it is desirable to design a toroidal field so that the relative variation of $U = \oint d\ell / B$ is small. This can be achieved by periodic bulges or scallops in the field, a kind of "bumpy torus" configuration.

An analytical study of this effect was made by considering the following problem: to determine a toroidal vacuum field, having closed lines, such that the variation of U is minimal, subject to certain conditions. These conditions are: (1) the minimum value of U in the torus is given; (2) $\oint B\, d\ell$ around the torus is given; (3) the torus is a periodic structure consisting of a given number n of identical sections in series.

The problem was solved for two-dimensional fields (meaning fields having no z-component and no z variation). The minimum variation of U for two-dimensional fields is approximately

$$\frac{\Delta U}{U} = \frac{U_{max} - U_{min}}{U_{min}} \sim 2\,(n-1)\,\left(\frac{a}{R}\right)^2$$

2D annulus instead of torus

where R, a are the major and minor radii of the torus.

In three dimensions a similar problem was solved in the large-n limit, assuming the field to have a special kind of dependence on n . The result here is that the minimum variation of U for the class of fields considered is

$$\frac{\Delta U}{U} \sim n\,\left(\frac{a}{R}\right)^2$$

This indicates an improvement of 50% over the two-dimensional value.

---

* This work accomplished under the auspices of the U. S. Atomic Energy Commission.

The Possible Value of Electron-Hole Plasma Experiments as Tests of Fusion Containment Schemes

J. E. Drummond and B. Ancker-Johnson

Boeing Scientific Research Laboratories

Kessler[1] recently called attention to the similarity between theory and experiment designed to determine the magnetic moment of an electron-hole plasma[2] to that concerned with the equilibrium of current-carrying toroidal electron-ion plasma (stellarator).[3] This work demonstrates the nearly complete correspondence between these classical diffusion processes. The modification of classical diffusion by instabilities and the control of this enhanced diffusion by minimum B magnetic fields has been studied in electron-ion plasmas by Ioffe et al.[4] and in electron-hole plasmas by Ancker-Johnson and Berg.[5] The stability theory by Furth and Rosenbluth[6] for maximum $\int dl/B$ is extended in the present work to show the correspondence between instability growth rates of fusion plasmas and measurable alteration of decay rates of electron-hole plasmas.

---

1. J. O. Kessler, Seminar on the Relations between Fusion and Solid-State Plasmas, Oak Ridge, March 29-30, 1965 (unpublished).
2. A. R. Moore and J. O. Kessler, Phys. Rev. 132, 1494 (1963).
3. S. Yoshikawa, R. M. Sinclair, J. O. Kessler, and W. L. Harries, Phys. Fluids 6, 932 (1963).
4. Iu. V. Gott, M. S. Ioffe, and V. G. Tel'kovskii, Nucl. Fusion Suppl., Pt. 3, 1045 (1963); Yu. T. Baiborodov, M. S. Ioffe, V. M. Petrov, and R. I. Sovolev, J. Nucl. Energy Pt. C, 5, 409 (1963).
5. B. Ancker-Johnson, Phys. Fluids 7, 1553 (1964); B. Ancker-Johnson, M. F. Berg, Proc. International Conf. on Phys. of Semiconductors, Paris, Dunod, 1964, p. 513.
6. H. P. Furth, Proc. International Symposium on Plasma Physics, Trieste, 1964 (to be published).

The Second Adiabatic Invariant in the Second Approximation

T. G. Northrop and C. S. Liu

Lawrence Radiation Laboratory
University of California, Livermore

Numerical integration of the guiding center trajectory in quadrupole mirror geometry has shown[1] deviation of the guiding center from surfaces which would be followed if the lowest order of the second invariant were conserved. These deviations occur principally near the axial currents where shear in the field is large. To understand this effect, the first correction to the second invariant is needed and has been obtained by the general theory of Kruskal.[2] The method of calculation will be reviewed and the result presented.

---

1. J. Siambis, Ph. D. Thesis, University of California, 1964.
2. M. D. Kruskal, J. Math. Phys. 3, 806 (1962).

Variational Principle for Long-Time Equilibrium in Particle Description*

M. Kruskal
Plasma Physics Laboratory, Princeton University

J. B. Taylor
Culham Laboratory

Of interest is a distribution of particles in a magnetic trap which is steady not only on the hydromagnetic time scale of particle mirror traversal but for the longer time scale of significant gradient-B drift. The class of such equilibria is studied by an adaptation of the thought-experiment approach used by Kruskal and Kulsrud for toroidal hydromagnetic equilibria. Particles are assigned to species according to their longitudinal (second) adiabatic invariant $J = \int v_{\parallel} \, d\ell$ in addition to the usual mass, charge, and (first invariant) magnetic moment $\mu = v_{\perp}^2/2B$. Constants of motion of the system under an artificially slow evolution are obtained--crucially, the third invariant (the flux enclosed by the magnetic cylinder traced out by the guiding center of a particle on the long time scale). It is shown that the variational conditions for the stationarity of the total energy of the system (under the constraints that these constants not vary) amount to just the conditions for long-time equilibrium. Possible equilibria are therefore characterizable by the sets of possible values of these constants in an assumed initial condition of the slow evolution. In the special case of equal-mass particles and no scalar potential, which is what has so far been completely analyzed, there is one equilibrium for each assignment of a single function of one variable to each species, this function specifying the number of particles per unit flux.

* This work accomplished under the auspices of the U. S. Atomic Energy Commission.

Influence of Wave Reflection on Mirror Confinement*

R. E. Aamodt and D. L. Book

General Atomic Division
General Dynamics Corporation

We discuss the effect of the reflection of growing modes from a density gradient on stability of confinement in a mirror geometry. A general convergent perturbation technique based on the WKB method is developed. It is applied to the fourth order differential equation which describes propagation of low frequency electrostatic waves in the direction of a density gradient along the field lines. As a result, we calculate the reflection coefficient and find that significant reflection can occur before Landau damping becomes important. When this result is applied to the loss cone instability, we find a critical length for mirror systems which is substantially shorter than that obtained by Rosenbluth and Post[1] in the neglect of reflection.

---

* Research sponsored by the Air Force Office of Scientific Research, Office of Aerospace Research, under Contract AF49(638)-1546.

1. M. N. Rosenbluth and R. F. Post, Phys. Fluids (~~to be published~~ most recent issue).

Stability of Mirror Machines

Harold Grad

Courant Institute of Mathematical Sciences
New York University

An analysis of interchange stability according to macroscopic and according to guiding center plasma theory indicates that there exists a large variety of interesting mirror-type plasma configurations which should be grossly stable. These configurations are easier to deal with theoretically and easier to build experimentally than most other "stabilized" versions of mirror machines.

$$\frac{\partial n_i'}{\partial t} - \frac{g}{\omega_c}\frac{\partial n_i'}{\partial y} + \frac{c E_{y1}}{B}\frac{\partial n_0}{\partial x} = 0$$

$$\frac{\partial n_e'}{\partial t} + c\frac{E_y'}{B}\frac{\partial n_0}{\partial x} - \frac{\sigma}{e}\frac{\partial E_{z1}}{\partial z} = 0$$

$$\frac{\omega}{\Omega_i} : \frac{g\, k_\perp}{\Omega_i^2} : \frac{c E_\perp}{B\, L_\perp}$$

$$\epsilon \sim \frac{g}{\Omega_i}\frac{k_\perp R_L}{\Omega_i R_L}\frac{v_i^2}{R_c L_\parallel \Omega_i} \sim \quad 1 \quad \frac{v_i^2}{\Omega_i^2}\frac{\cancel{L_\perp}}{R_c}\frac{\cancel{L_\perp}}{\cancel{L_\parallel}}\frac{\epsilon^2}{L_\perp}$$

$$g \sim \frac{v_i^2}{2 R_c}$$

Instabilities of the "Low-Density Stable Regime" in the Mirror Machine*

Harold P. Furth

Lawrence Radiation Laboratory
University of California, Livermore

For hot-ion, low-density mirror machine experiments[1] the simple flute instability is expected to disappear[2] when $k^2 D^2 > 4 k_o R_c$, where D is the Debye length, R_c the field-line curvature, $k_o = n'/n$, and k is the wave number. The observed persistence of "stable" oscillations[1] suggests that in the linear analysis the flute may remain weakly unstable even in the low-density regime. For the simple model of a monoenergetic ion distribution, one finds that a weak dissipative mechanism, for example resistive line-tying, will excite the upper branch[1] (near the ion precession frequency) of "stable" oscillation, and damp the lower branch; but a strong dissipation will tend to suppress both branches. When $k^2 D^2 >> 1$, the magnitude of the maximum growth rate is $\omega_g = \omega_c (k_o/k)(r_c^2/D^2)$ where ω_c and r_c are the frequency and radius of gyration. In minimum-B geometry, both branches are always damped. The occasional experimental observation, in mirror geometry, of the lower branch mode (apparently depending on particle velocity distributions) suggests that a complete explanation should include finite-beam-spread effects. I am indebted to M. N. Rosenbluth[3] for helpful discussions.

$k_o R_c < 0$ for mirror

---

* Work performed under the auspices of the U. S. Atomic Energy Commission.

1. C. C. Damm, J. H. Foote, A. H. Futch, and R. F. Post, Phys. Rev. Letters 10, 323 (1963).

2. B. Lehnert, Nuclear Fusion, 1962 Supplement, Part 1, p. 135.

3. D. B. Chang, L. D. Pearlstein, and M. N. Rosenbluth, "On the Interchange Stability of the Van Allen Belt", GA-6007 (January 1965).

Stability of Flute and Universal Modes in a Finite Pressure Plasma*

Nicholas A. Krall and L. Donald Pearlstein

General Atomic Division
General Dynamics Corporation

Most of the familiar drift instabilities (flute, universal, etc.) were calculated in the limit β = (plasma pressure)/(magnetic pressure) = 0. In this limit the perturbed electric field is electrostatic, and the Vlasov equation is remarkably simplified. The β limit for validity of the electrostatic approximation depends on the particular mode considered and many present experiments fall outside the range. Two well-known low frequency drift instabilities are examined at arbitrary β; the zero β regime is determined and stability criteria for larger values of β are derived. The finite Larmor radius (R) flute instability in a uniform magnetic field $\vec{B} = B_o \hat{i}_{\parallel}$ simulating field curvature by a fictitious gravity is considered first. It is shown that the lowest order ominously large contribution, inversely proportional to R^2, cancels to all orders of β. The leading terms are then independent of R, and are included in the eigenvalue problem; stability criteria are obtained for various ranges of β. Next the universal instability $\vec{E}_{\text{perturbed}} = \vec{E} e^{ik_\perp r_\perp} e^{ik_\parallel r_\parallel}$ is considered. Here the zero β limit is dependent upon wavelengths. If $\beta <$ (electron mass)/(ion mass), or if $\beta < (k_\parallel r_p)^2$ where r_p = plasma radius and $1/k_\parallel \geq$ plasma length, the zero β limit obtains. Since the instability only exists for $(k_\parallel r_p)^2 < 0.01$, a modest plasma pressure violates the zero β limit.

We include finite $\beta (k_\parallel r_p)^{-2}$, and calculate finite length stabilization, with and without magnetic curvature. We find that the other effect of curvature, stabilization by cusps of arbitrary length systems, is unaffected by finite β.

---

* This work was supported in part under a joint General Atomic-Texas Atomic Energy Research Foundation program on Controlled Thermonuclear Reactions, and in part by the National Aeronautics and Space Administration, Office of Space Sciences, Geophysics and Astronomy Program.

Flute Instabilities at Ion Gyrofrequency*

T. K. Fowler and Y. Shima

Oak Ridge National Laboratory

Flute modes propagating perpendicular to B are expected to be important sources of anomalous diffusion. Besides hydromagnetic flutes, there also exist flute modes at the ion gyrofrequency ω_{ci} and its harmonics caused by resonance between ion gyration and various drift waves, notably the diamagnetic drift proportional to the gradient of the density n_o. In their investigation of these "drift-cyclotron" modes, Mikhailovsky and Timofeev[1] found a density threshold for instability at $\omega_{pi} = (R/a)\,\omega_{ci}$ where $R = (n_o^{-1} dn_o/dr^2)^{-\frac{1}{2}}$ for cylinders, a is the ion gyroradius and ω_{pi} is the ion plasma frequency. In large plasmas, this corresponds to high densities, for example $n = 10^{14}$/cc at $R/a = 100$ and $B = 10$ kg. Maxwellian ions and a monotonically decreasing density were assumed. In search of gyrofrequency flutes at lower densities, we have examined various non-Maxwellian ion distributions (as in mirror machines) and radial density $n_o \propto r^2$ at $r = 0$. For the latter, the threshold is $\omega_{pi} \sim (R/a)^{\frac{1}{2}}\,\omega_{ci}$, considerably lower than that for the case considered by Mikhailovsky. Throughout we employ cylindrical geometry rather than the slab approximation used by Mikhailovsky. We confirm results for his case.

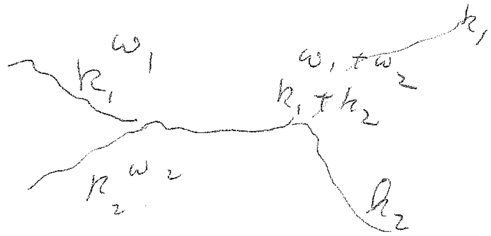
---

* Research sponsored by the U.S. Atomic Energy Commission under contract with the Union Carbide Corporation.

1. A. B. Mikhailovsky and A. V. Timofeev, Soviet Phys.—JETP 17, 626 (1963).

in either homogeneous or inhomogeneous plasmas. Drummond

resonant mode coupling if it's a freq. plasma generates



nonlinear Landau damping $\quad \dfrac{\omega_1 \mp \omega_2}{k_1 \mp k_2} \ll \bar{v}$ slow for one dimension, not so for 3-D

Three wave resonance property conserves energy $\dfrac{\partial}{\partial t}\sum |E_k|^2 = 0$

Four wave interaction $\quad \dfrac{\partial}{\partial t} \sum_k \dfrac{|E_k|^2}{\omega} = 0$

$$\frac{|E_k|^2}{\omega} = N_k \quad \text{(quasi-particles}$$

scattering of quasi-particles by electrons or ions $\quad \sum N_k = \text{constant}$

waves & particles conserved (?)

Rudaker

$$\frac{\partial N}{\partial t} + \frac{\partial N}{\partial x}\frac{\partial \omega}{\partial k} - \frac{\partial N}{\partial k}\frac{\partial \omega}{\partial x} = 0$$

obeys cons. law along some path in x, k space

Background plasma sees force due to gradient of radiation pressure in addition to usual forces (needed for electrons but not needed in ions)

if plasma has well-developed μ-turbulence then particles collide with quasi-particles even if "collisionless" — turbulent resistance

spectrum of turbulence (maybe Langmuir oscillation)

$$\frac{\partial f_0}{\partial t} + v\frac{\partial f}{\partial x} - \frac{eE^0}{m}\frac{\partial f_0}{\partial v} + \frac{()}{m}\nabla \sum \frac{|E_k|^2}{(\omega_n)^2}\frac{\partial f}{\partial v} = 0$$

trajectory of quasi-particles

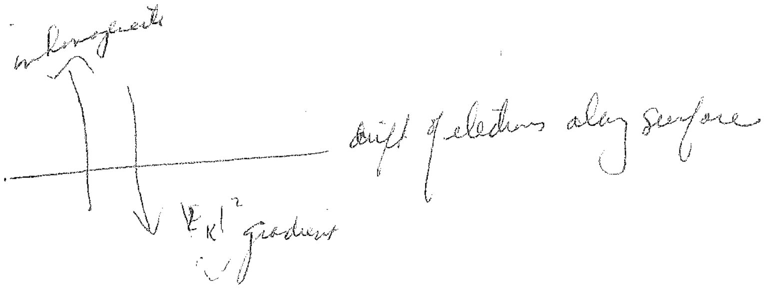
$\Rightarrow$ joule heating of electrons to several kilovolts

Ion Waves in Bounded Plasma

B. Bertotti, A. Cavaliere and P. Guipponi

Laboratorio Gas Ionizzati, Frascati

The role of the charge sheath at the wall of a discharge in determining the appropriate boundary conditions for a low frequency wave in a low pressure discharge is discussed by means of a hydrodynamical model. It is found that Bohm's condition on the mean ion velocity holds even when oscillations are present. The electron dynamics is neglected and the dispersion relation shows the existence of a principal mode which propagates along the tube just like in an infinite medium; all other modes are damped. Analysis of the dynamics of the oscillating sheath shows that it behaves like a battery, transferring the electrostatic oscillation to the wall with no distortion and no impedance.



effective collision ~~time~~ frequency can go as high as ω_{pe} !! - Rudakov

Shock forms in K space - Rankine Hugoniot - jump in K space

where WKB violated -

diffusion rates $\rightarrow$ Bohm diffusion

Steady Nonlinear Waves in a Warm
Collision-free Plasma

H. Kever and G. Morikawa

Courant Institute of Mathematical Sciences
New York University

We give a self-consistent formulation describing the steady motion of a warm, weak plasma pulse moving oblique to a strong magnetic field. A rather complex parametric relation is obtained connecting the pulse width to the other physical parameters, i.e., mass ratio of electrons and ions, obliqueness angle, and temperature of electrons and ions.

Ion-Acoustic Waves and Electron Plasma Oscillations

H. Weitzner and D. Dobrott

Courant Institute of Mathematical Sciences
New York University

In certain parameter ranges it is possible to observe exponential damping of waves in a collisionless plasma for a limited interval in space or time. Results are presented both for ion-acoustic waves and for ordinary electron plasma oscillations. No assumptions other than smoothness are made concerning the various distribution functions, and under appropriate circumstances Maxwellians are acceptable distribution functions.

Structure of a Perpendicular Shock Wave*

Pung Nien Hu**

Courant Institute of Mathematical Sciences
New York University

Based on the moment equations of the Boltzmann equations for fully ionized gases, analytic solutions of weak perpendicular shock waves are obtained. The result shows that all weak shock waves, in ionized gases as well as in neutral gases, have similar profiles given by the hyperbolic-tangent function of the distance. The shock thickness is found depending on four basic parameters, namely, the mean free path of the gas, the length of penetration, the Larmor radius based on thermal speed, and the mass ratio of electron and ion. From the explicit expression of the shock thickness, various significant cases are derived.

The most interesting case occurs when the mean free path is larger than the electron length of penetration which is in turn larger than the electron Larmor radius; the shock thickness is always less than the electron length of penetration if the latter is ~~less~~ larger than the geometric mean of the mean free path and the electron Larmor radius. Furthermore, if the mean free path is in the order of the geometric mean of the electron and ion lengths of penetration, the shock thickness is in the order of the geometric mean of the electron and ion Larmor radii.

ϵ = scaled shock thickness

$\epsilon = \dfrac{u-u'}{2U}$

---

* Work supported by the Atomic Energy Commission.

** Present address: Davidson Laboratory, Stevens Institute of Technology, Hoboken, New Jersey.

can close eqs. just due to scaling $x = \epsilon y$

$H \uparrow$ (shock transition region) $\uparrow H'$

$\overrightarrow{u}$ $\overrightarrow{u'}$

Radiation by a Plasma:
Quadrupole Bremsstrahlung and Synchrotron Emission*

Thomas J. Birmingham

Plasma Physics Laboratory, Princeton University

A model previously developed[1] for computing the radiative emission from a plasma has been applied to two new problems. The long wavelength expansion previously considered only to lowest (dipole) order has been extended and the power spectra resulting from quadrupole electron-ion and electron-electron collisions evaluated. Even for relativistically energetic plasma, the contribution due to these effects is found to be small by comparison with the dominant electron-ion dipole emission. The generation of synchrotron radiation by relativistically gyrating electrons has also been computed within the framework of the model. In the absence of a plasma background, the Trubnikov[2] power spectrum is obtained. In addition, corrections of order $(\omega_p/\omega)^2$ are derived for the emission at higher harmonics by a relativistic electron radiating into a colder plasma. Numerical calculations indicate that the effect of the plasma is to reduce the power radiated.

---

* This work accomplished under the auspices of the U. S. Atomic Energy Commission.

1. T. Birmingham, J. Dawson, and C. Oberman, Phys. Fluids 8, 297 (1965).

2. B. A. Trubnikov, Ph. D. Thesis, Moscow Institute of Engineering and Physics, 1958 (Translation: AEC-tr-4073).

The Diffusion of Test Particles in a Magnetic Field*

O. C. Eldridge and E. G. Harris

University of Tennessee

The diffusion coefficients for the transport of a plasma across a magnetic field are calculated directly from the motion of test particles. The method is sufficiently general to describe anomalous diffusion as well as collisional diffusion. A consistent perturbation theory is developed for calculating particle orbits in which secular terms are eliminated. These secular terms, quantities that increase linearly with time, appear when conventional perturbation theory is applied to the orbits and invalidates the basis of the theory.

The spatial diffusion coefficient is found to be directly related to the diffusion in velocity space. An equation is developed which describes the diffusion in phase space, both spatial and velocity diffusion. The diffusion coefficients are derived for an arbitrary spectrum of fluctuations of the electric field. The electric field is assumed to be longitudinal, but no restrictions are imposed on its frequency or wavelength.

---

* This work was supported in part by a consulting contract with the Oak Ridge National Laboratory.